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Heterotic strings from matrices

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Abstract: We propose a nonperturbative definition of heterotic string theory on arbitrary multidimensional tori.

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Contents

1 Introduction

The matrix model of uncompactified M theory [1]-[3] has been generalized to arbitrary toroidal compactifications of type IIA and IIB string theory. These models can be viewed as particular large M limits of the original matrix model, in the sense that they may be viewed as the dynamics of a restricted class of large M matrices, with the original matrix model Lagrangian.

A separate line of reasoning has led to a description of the Hořava-Witten domain wall in terms of matrix quantum mechanics [4]. Here, extra degrees of freedom have to be added to the original matrix model. As we will review below, if these new variables, which tranform in the vector representation of the gauge group, are not added, then the model does not live in an eleven dimensional spacetime, but only on its boundary.. Although it is, by construction, a unitary quantum mechanics, it probably does not recover ten dimensional Lorentz invariance in the large M limit. Its nominal massless particle content is the ten dimensional $N = 1$ supergravity (SUGRA) multiplet, which is anomalous.

With the proper number of vector variables added, the theory does have an eleven dimensional interpretation. It is possible to speak of states far from the domain wall and to show that they behave exactly like the model of [3]. Our purpose in the present paper is to compactify this model on spaces of the general form $S^1/Z_2 \times T^d$. We begin by reviewing the argument for the single domain wall quantum mechanics, and generalize it to an S^1/Z_2 compactification. The infinite momentum frame Hamiltonian for this system is practically identical to the static gauge $O(M)$ Super Yang Mills (SYM) Hamiltonian for M heterotic D strings in Type I string theory. They differ only in the boundary conditions imposed on the fermions which transform in the vector of $O(M)$. These fermions are required for $O(M)$ anomaly cancellation in both models, but the local anomaly does not fix their boundary conditions. Along the moduli space of the $O(M)$ theory, the model exactly reproduces the string field theory Fock space of heterotic string theory. The inclusion of both Ramond and Neveu Schwarz boundary conditions for the matter fermions, and the GSO projection, are simple consequences of the $O(M)$ gauge invariance of the model.

Generalizing to higher dimensions, we find that the heterotic matrix model on $S^1/Z_2 \times T^d$ is represented by a $U(M)$ gauge theory on $S^1 \times T^d/Z_2$. On the orbifold circles, the gauge invariance reduces to $O(M)$. We are able to construct both the heterotic and open string sectors of the model, which dominate in different limits of the space of compactifications.

In the conclusions, we discuss the question of whether the heterotic models which we have constructed are continuously connected to the original uncompactified eleven dimensional matrix model. The answer to this question leads to rather surprising conclusions, which inspire us to propose a conjecture about the way in which the matrix model solves the cosmological constant problem. It also suggests that string vacua with different numbers of supersymmetries are really states of *different* underlying theories. They can only be continuously connected in limiting situations where the degrees of freedom which differentiate them decouple.

2 Heterotic matrix models in ten and eleven dimensions

In [5] an $O(M)$ gauged supersymmetric matrix model for a single Hořava-Witten domain wall embedded in eleven dimensions was proposed. It was based on an extrapolation of the quantum mechanics describing $D0$ branes near an orientifold plane in Type IA string theory [6]. The model was presented as an orbifold of the original [3] matrix model in [7]. In the Type IA context it is natural to add degrees of freedom transforming in the vector of $O(M)$ and corresponding to the existence of $D8$ branes and the 08 strings connecting them to the $D0$ branes. Since $D8$ branes are movable in Type IA theory, there are consistent theories both with and without these extra degrees of freedom. That is, we can consistently give them masses, which represent the distances between the $D8$ branes and the orientifold. However, as first pointed out by [6], unless the number of $D8$ branes sitting near the orientifold is exactly 8, the $D0$ branes feel a linear potential which either attracts them to or repels them from the orientifold. This is the expression in the quantum mechanical approximation, of the linearly varying dilaton first found by Polchinski and Witten [8]. This system was studied further in [9] and [10]. In the latter work the supersymmetry and gauge structure of model were clarified, and the linear potential was shown to correspond to the fact that the "supersymmetric ground state" of the model along classical flat directions representing excursions away from the orientifold was not gauge invariant.

From this discussion it is clear that the only way to obtain a model with an eleven dimensional interpretation is to add sixteen massless real fermions transforming in the vector of $O(M)$, which is the model proposed in [5]. In this case, $D0$ branes can move freely away from the wall, and far away from it the theory reduces to the $U([\frac{M}{2}])$ model of [3].[1]

Our task now is to construct a model representing two Hořava-Witten end of the world 9-branes separated by an interval of ten dimensional space. As in [7] we can approach this task by attempting to mod out the $1+1$ dimensional field theory [3], [14], [15], [11], [16] which describes M theory compactified on a circle. Following the logic of [7], this leads to an $O(M)$ gauge theory. The 9-branes are stretched around the longitudinal direction of the infinite momentum frame (IMF) and the $2-9$ hyperplane of the transverse space. X^1 is the differential operator

$$\frac{R_1}{i}\frac{\partial}{\partial\sigma} - A_1\,,$$

where σ is in $[0, 2\pi]$, and A_1 is an $O(M)$ vector potential. The other X^i transform in the $\mathbf{\frac{M(M+1)}{2}}$ of $O(M)$. There are two kinds of fermion multiplet. θ is an $\mathbf{8_c}$ of the spacetime $SO(8)$, a symmetric tensor of $O(M)$ and is the superpartner of X^i under the eight dynamical and eight kinematical SUSYs which survive the projection. λ is in the adjoint of $O(M)$, the $\mathbf{8_s}$ of $SO(8)$, and is the superpartner of the gauge potential. We will call it the gaugino.

As pointed out in [9] and [10], this model is anomalous. One must add 32 Majorana-Weyl fermions χ in the $\mathbf{M}$ of $O(M)$. For sufficiently large M, this is the only fermion content which can cancel the anomaly. The continuous $SO(M)$ anomaly does not fix the boundary conditions of the χ fields. There are various consistency conditions which help to fix them, but in part we must make a choice which reflects the physics of the situation which we are trying to model.

The first condition follows from the fact that our gauge group is $O(M)$ rather than $SO(M)$. That is, it should consist of the subgroup of $U(M)$ which survives the orbifold projection. The additional Z_2 acts only on the χ fields, by reflection. As a consequence, the general principles of gauge theory tell us that each χ field might appear with either periodic or antiperiodic boundary conditions, corresponding to a choice of $O(M)$ bundle. We must also make a projection by the discrete transformation which reflects all the χ's. What is left undetermined by these principles is choice of relative boundary conditions among the 32 χ's.

The Lagrangian for the χ fields is

$$\chi(\partial_t + 2\pi R_1\partial_\sigma - iA_0 - iA_1)\chi. \tag{2.1}$$

In the large R_1 limit, the volume of the space on which the gauge theory is compactified is small, and its coupling is weak, so we can treat it by semiclassical methods. In particular,

[1] Actually there is a highly nontrivial question which must be answered in order to prove that the effects of the wall are localized. In [10] it was shown that supersymmetry allowed an arbitary metric for the coordinate representing excursions away from the wall. In finite orders of perturbation theory the metric falls off with distance but, as in the discussion of the graviton scattering amplitude in [3], one might worry that at large M these could sum up to something with different distance dependence. In [3] a nonrenormalization theorem was conjectured to protect the relevant term in the effective action. This cannot be the case here.

the Wilson lines become classical variables. We will refer to classical values of the Wilson lines as expectation values of the gauge potential A_1. (We use the term expectation value loosely, for we are dealing with a quantum system in finite volume. What we mean is that these "expectation values" are the slow variables in a system which is being treated by the Born-Oppenheimer approximation.) An excitation of the system at some position in the direction tranverse to the walls is represented by a wave function of $n \times n$ block matrices in which A_1 has an expectation value breaking $O(n)$ to $U(1) \times U([n/2])$. In the presence of a generic expectation value, in $A_0 = 0$ gauge, the χ fields will not have any zero frequency modes. The exceptional positions where zero frequency modes exist are $A_1 = 0$ (for periodic fermions) and $A_1 = \pi R_1$ (for antiperiodic fermions). These define the positions of the end of the world 9-branes, which we call the walls. When $R_1 \gg l_{11}$, all of the finite wavelength modes of all of the fields have very high frequencies and can be integrated out. In this limit, an excitation far removed from the walls has precisely the degrees of freedom of a $U([\frac{n}{2}])$ gauge quantum mechanics. The entire content of the theory far from the walls is $U([\frac{M}{2}])$ gauge quantum mechanics. It has no excitations carrying the quantum numbers created by the χ fields, and according to the conjecture of [3] it reduces to eleven dimensional M theory in the large M limit. This reduction assumes that there is no longe range interaction between the walls and the rest of the system.

In order to fulfill this latter condition it must be true that at $A_1 = 0$, and in the large R_1 limit, the field theory reproduces the $O(M)$ quantum mechanics described at the beginning of this section (and a similar condition near the other boundary). We should find 16 χ zero modes near each wall. *Thus, the theory must contain a sector in which the* 32 $1+1$ *dimensional* χ *fields are grouped in groups of* 16 *with opposite periodicity.* Half of the fields will supply the required zero modes near each of the walls. Of course, the question of which fields have periodic and which antiperiodic boundary conditions is a choice of $O(M)$ gauge. However, in any gauge only half of the χ fields will have zero modes located at any given wall. We could of course consider sectors of the fundamental $O(M)$ gauge theory in which there is a different correlation between boundary conditions of the χ fields. However, these would not have an eleven dimensional interpretation at large R_1. The different sectors are not mixed by the Hamiltonian so we may as well ignore them.

To summarize, we propose that M theory compactified on S^1/Z_2 is described by a $1+1$ dimensional $O(M)$ gauge theory with $(0, 8)$ SUSY. Apart from the (A_μ, λ) gauge multiplet, it contains a right moving X^i, θ supermultiplet in the symmetric tensor of $O(M)$ and 32 left moving fermions, χ, in the vector. The allowed gauge bundles for χ (which transforms under the discrete reflection which leaves all other multiplets invariant), are those in which two groups of 16 fields have opposite periodicities. In the next section we will generalize this construction to compactifications on general tori.

First let us see how heterotic strings emerge from this formalism in the limit of small R_1. It is obvious that in this limit, the string tension term in the SYM Lagrangian becomes very small. Let us rescale our X^i and time variables so that the quadratic part of the Lagrangian is independent of R_1. Then, as in [11], [15], [16], the commutator term involving the X^i gets a coefficient R^{-3} so that we are forced onto the moduli space in that limit. In this $O(M)$ system, this means that the X^i matrices are diagonal, and the gauge group is completely broken to a semidirect product of Z_2 (or $O(1)$) subgroups which reflect the individual components of the vector representation, and an S_M which permutes the eigenvalues of the X^i. The moduli space of low energy fields[2] consists of diagonal X^i fields, their superpartners θ_a (also diagonal matrices), and the 32 massless left moving χ fields. The gauge bosons and their superpartners $\lambda^{\dot{\alpha}}$ decouple in the small R_1 limit. All of the χ fields are light in this limit.

2.1 Screwing heterotic strings

As first explained in [15] and elaborated in [11], and [16], twisted sectors under S_N lead to strings of arbitrary length.[3] The strings of conventional string theory, carrying continuous values of the longitudinal momentum, are obtained by taking N to infinity and concentrating on cycles whose length is a finite fraction of N. The new feature which arises in the heterotic string is that the boundary conditions of the χ fields can be twisted by the discrete group of reflections.

A string configuration of length $2\pi k$, $X^i_S(s)$, $0 \leq s \leq 2\pi k$, is represented by a diagonal matrix:

$$X^i(\sigma) = \begin{pmatrix} X^i_S(\sigma) & & & \\ & X^i_S(\sigma + 2\pi) & & \\ & & \ddots & \\ & & & X^i_S(\sigma + 2\pi(N-1)) \end{pmatrix}. \tag{2.2}$$

This satisfies the twisted boundary condition $X^i(\sigma + 2\pi) = E_O^{-1} X^i(\sigma) E_O$ with

$$E_O = \begin{pmatrix} & & & & \epsilon_k \\ \epsilon_1 & & & & \\ & \epsilon_2 & & & \\ & & \ddots & & \\ & & & \epsilon_{N-1} & \end{pmatrix}, \tag{2.3}$$

and $\epsilon_i = \pm 1$. The latter represent the $O(1)^k$ transformations, which of course do not effect X^i at all.

To describe the possible twisted sectors of the matter fermions we introduce the matrix $r^a_b = diag(1 \dots 1, -1 \dots -1)$, which acts on the 32 valued index of the χ fields. The sectors

[2]We use the term moduli space to refer to the space of low energy fields whose effective theory describes the small R_1 limit (or to the target space of this effective theory). These fields are in a Kosterlitz Thouless phase and do not have expectation values, but the term moduli space is a convenient shorthand for this subspace of the full space of degrees of freedom.

[3]These observations are mathematically identical to considerations that arose in the counting of BPS states in black hole physics [12].

are then defined by

$$\chi^a(\sigma + 2\pi) = r^a_b E_O^{-1} \chi^b(\sigma)\,. \tag{2.4}$$

As usual, inequivalent sectors correspond to conjugacy classes of the gauge group. In this case, the classes can be described by a permutation with a given set of cycle lengths, corresponding to a collection of strings with fixed longitudinal momentum fractions, and the determinants of the $O(1)^k$ matrices inside each cycle. In order to understand the various gauge bundles, it is convenient to write the "screwing formulae" which express the components of the vectors χ^a in terms of string fields χ^a_s defined on the interval $[0, 2\pi k]$. The defining boundary conditions are

$$\chi^a_i(\sigma + 2\pi) = \epsilon_i r^a_b \chi^b_{i+1}(\sigma)\,, \tag{2.5}$$

where we choose the gauge in which $\epsilon_{i<k} = 1$ and $\epsilon_k = \pm 1$ depending on the sign of the determinant. The vector index i is counted modulo k. This condition is solved by

$$\chi^a_i(\sigma) = (r^{i-1})^a_b \chi^b_S(\sigma + 2\pi(i-1))\,, \tag{2.6}$$

where χ_S satisfies

$$\chi^a_S(\sigma + 2\pi k) = (r^k)^a_b \epsilon_k \chi^b_S(\sigma)\,. \tag{2.7}$$

For k even, this gives the PP and AA sectors of the heterotic string, according to the sign of the determinant. Similarly, for k odd, we obtain the AP and PA sectors.

As usual in a gauge theory, we must project on gauge invariant states. It turns out that there are only two independent kinds of conditions which must be imposed. In a sector characterized by a permutation S, one can be chosen to be the overall multiplication of χ fields associated with a given cycle of the permutation (a given string) by -1. This GSO operator anticommuting with all the 32 χ fields is represented by the $-\mathbf{1}$ matrix from the gauge group $O(N)$. The other is the projection associated with the cyclic permutations themselves. It is easy to verify that under the latter transformation the χ_S fields transform as

$$\chi^a_S(\sigma) \to r^a_b \chi^b_S(\sigma + 2\pi)\,. \tag{2.8}$$

Here $\sigma \in [0, 2\pi k]$ and we are taking the limit $M \to \infty$, k/M fixed. In this limit the 2π shift in argument on the righthand side of (2.8) is negligible, and we obtain the second GSO projection of the heterotic string.

Thus, $1+1$ dimensional $O(M)$ SYM theory with $(0, 8)$ SUSY, a left moving supermultiplet in the symmetric tensor representation and 32 right moving fermion multiplets in the vector (half with P and half with A boundary conditions) reduces in the weak coupling, small (dual) circle limit to two copies of the Hořava-Witten domain wall quantum mechanics, and in the strong coupling large (dual) circle limit, to the string field theory of the $E_8 \times E_8$ heterotic string.

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3 Multidimensional cylinders

The new feature of heterotic compactification on $S^1/Z_2 \times T^d$ is that the coordinates in the toroidal dimensions are represented by covariant derivative operators with respect to new world volume coordinates. We will reserve σ for the periodic coordinate dual to the interval S^1/Z_2 and denote the other coordinates by σ^A. Then,

$$X^A = \frac{2\pi R_A}{i} \frac{\partial}{\partial \sigma^A} - A_A(\sigma); \quad A = 2 \ldots k+1 \,. \tag{3.1}$$

Derivative operators are antisymmetric, so in order to implement the orbifold projection, we have to include the transformation $\sigma^A \to -\sigma^A$, for $A = 2 \ldots d+1$, in the definition of the orbifold symmetry. Thus, the space on which SYM is compactified is $S^1 \times (T^d/Z_2)$. There are 2^d *orbifold circles* in this space, which are the fixed manifolds of the reflection. Away from these singular loci, the gauge group is $U(M)$ but it will be restricted to $O(M)$ at the singularities. We will argue that there must be a number of $1+1$ dimensional fermions living only on these circles. When $d=1$ these orbifold lines can be thought of as the boundaries of a *dual cylinder*. Note that if we take $d=1$ and rescale the σ^A coordinates so that their lengths are $1/R_A$ then a long thin cylinder in spacetime maps into a long thin cylinder on the world volume, and a short fat cylinder maps into a short fat cylinder. As we will see, this geometrical fact is responsible for the emergence of Type IA and heterotic strings in the appropriate limits.

The boundary conditions on the world volume fields are

$$X^i(\sigma, \sigma^A) = \bar{X}^i(\sigma, -\sigma^A)\,, \quad A_a(\sigma, \sigma^A) = \bar{A}_a(\sigma, -\sigma^A)\,, \tag{3.2}$$

$$A_1(\sigma, \sigma^A) = -\bar{A}_1(\sigma, -\sigma^A)\,, \tag{3.3}$$

$$\theta(\sigma, \sigma^A) = \bar{\theta}(\sigma, -\sigma^A)\,, \quad \lambda(\sigma, \sigma^A) = -\bar{\lambda}(\sigma, -\sigma^A)\,. \tag{3.4}$$

All matrices are hermitian, so transposition is equivalent to complex conjugation. The right hand side of the boundary condition 3.3 can also be shifted by $2\pi R_1$, reflecting the fact that A_1 is an angle variable.

Let us concentrate on the cylinder case, $d=1$. In the limits $R_1 \ll l_{11} \ll R_2$ and $R_2 \ll l_{11} \ll R_1$, we will find that the low energy dynamics is completely described in terms of the moduli space, which consists of commuting X^i fields. In the first of these limits, low energy fields have no σ^2 dependence, and the boundary conditions restrict the gauge group to be $O(M)$, and force X^i and θ to be real symmetric matrices. Anomaly arguments then inform us of the existence of 32 fermions living on the boundary circles. The model reduces to the $E_8 \times E_8$ heterotic matrix model described in the previous section, which, in the indicated limit, was shown to be the free string field theory of heterotic strings.

3.1 Type IA strings

The alternate limit produces something novel. Now, low energy fields are restricted to be functions only of σ^2. Let us begin with a description of closed strings. We will exhibit a solution of the boundary conditions for each closed string field $X_S(\sigma)$ with periodicity $2\pi k$. Multiple closed strings are constructed via the usual block diagonal procedure:

$$X^i(\sigma^2) = U(\sigma^2) D U^{-1}(\sigma^2)\,, \tag{3.5}$$

$$\begin{aligned} D = \operatorname{diag}\big(&X^i_s(\sigma^2), \epsilon X^i_s(2\pi-\sigma^2), X^i_s(2\pi+\sigma^2), \epsilon X^i_s(4\pi-\sigma^2), \dots \\ &\dots, X^i_s(2\pi(N-1)+\sigma^2), \epsilon X^i_s(2\pi N-\sigma^2)\big)\ , \end{aligned} \tag{3.6}$$

where ϵ is $+1$ for $X^{2\dots9}$ and θ's, -1 for A^1 and λ's. From this form it is clear that the matrices will commute with each other for any value of σ^2. We must obey Neumann boundary conditions for the real part of matrices and Dirichlet conditions for the imaginary parts (or for $\epsilon=-1$ vice versa), so we must use specific values of the unitary matrix $U(\sigma^2)$ at the points $\sigma^2 = 0, \pi$. Let us choose

$$U'(0+) = U'(\pi-) = 0 \tag{3.7}$$

(for instance, put U constant on a neighbourhood of the points $\sigma^2 = 0, \pi$) and for a closed string:

$$U(\pi) = \begin{pmatrix} m & & & \\ & m & & \\ & & \ddots & \\ & & & m \end{pmatrix}, \qquad U(0) = C\cdot U(\pi)\cdot C^{-1} \tag{3.8}$$

where m are 2×2 blocks (there are N of them) and C is a cyclic permutation matrix:

$$C = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ 1 & & & & \end{pmatrix}. \tag{3.9}$$

Here the 1's are 1×1 matrices so that we have a shift of the $U(\pi)$ along the diagonal by half the size of the Pauli matrices.

The form of these blocks guarantees the conversion of τ_3 to τ_2:

$$m = \frac{\tau_2 + \tau_3}{\sqrt{2}}\,. \tag{3.10}$$

This 2×2 matrix causes two ends to be connected on the boundary. It is easy to check that the right boundary conditions will be obeyed.

To obtain open strings, we just change the $U(0)$ and $U(\pi)$. An open string of odd length is obtained by throwing out the last element in (3.6) and taking

$$U(0) = \begin{pmatrix} 1_{1\times1} & & & & \\ & m & & & \\ & & m & & \\ & & & \dots & \\ & & & & m \end{pmatrix}, \quad U(\pi) = \begin{pmatrix} m & & & & \\ & m & & & \\ & & m & & \\ & & & \dots & \\ & & & & 1_{1\times1} \end{pmatrix}. \tag{3.11}$$

Similarly, an open string of even length will have one of the matrices $U(0), U(\pi)$ equal to what it was in the closed string case $m \otimes 1$ while the other will be equal to

$$U(0) = \begin{pmatrix} 1_{1\times1} & & & & & \\ & m & & & & \\ & & m & & & \\ & & & \cdots & & \\ & & & & m & \\ & & & & & 1_{1\times1} \end{pmatrix}. \tag{3.12}$$

Similar constructions for the fermionic coordinates are straightforward to obtain. We also note that we have worked above with the original boundary conditions and thus obtain only open strings whose ends are attached to the wall at $R_1 = 0$. Shifting the boundary condition 3.3 by $2\pi R_1$ (either at $\sigma^2 = 0$ or $\sigma^2 = \pi$ or both) we obtain strings attached to the other wall, or with one end on each wall. Finally, we note that we can perform the gauge transformation $M \to \tau_3 M \tau_3$ on our construction. This has the effect of reversing the orientation of the string fields, $X_S(\sigma^2) \to X_S(-\sigma^2)$. Thus we obtain unoriented strings.

We will end this section with a brief comment about moving $D8$ branes away from the orientifold wall. This is achieved by adding explicit $SO(16) \times SO(16)$ Wilson lines to the Lagrangian of the χ^a fields. We are working in the regime $R_2 \ll l_{11} \ll R_1$, and we take these to be constant gauge potentials of the form $\chi^a \mathcal{A}_{ab} \chi^b$, with $\mathcal{A}$ of order R_1. In the presence of such terms χ^a will not have any low frequency modes, unless we also shift the $O(M)$ gauge potential A_1 to give a compensating shift of the χ frequency. In this way we can construct open strings whose ends lie on $D8$ branes which are not sitting on the orientifold.

In this construction, it is natural to imagine that 16 of the χ fields live on each of the boundaries of the dual cylinder. Similarly, for larger values of d it is natural to put $\frac{32}{2^d}$ fermions on each orbifold circle, a prescription which clearly runs into problems when $d > 4$. This is reminiscent of other orbifold constructions in M theory in which the most symmetrical treatment of fixed points is not possible (but here our orbifold is in the dual world volume). It is clear that our understanding of the heterotic matrix model for general d is as yet quite incomplete. We hope to return to it in a future paper.

4 Conclusions

We have described a class of matrix field theories which incorporate the Fock spaces of the the $E_8 \times E_8$ heterotic/Type IA string field theories into a unified quantum theory. The underlying gauge dynamics provides a prescription for string interactions. It is natural to ask what the connection is between this nonperturbatively defined system and previous descriptions of the nonperturbative dynamics of string theories with twice as much supersymmetry. Can these be viewed as two classes of vacua of a single theory?

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Can all of these be obtained as different large N limits of a quantum system with a finite number of degrees of freedom?

The necessity of introducing the χ fields into our model suggests that the original eleven dimensional system does not have all the necessary ingredients to be the underlying theory. Yet we are used to thinking of obtaining lower dimensional compactifications by restricting the degrees of freedom of a higher dimensional theory in various ways. Insight into this puzzle can be gained by considering the limit of heterotic string theory which, according to string duality, is supposed to reproduce M theory on $K3$. The latter theory surely reduces to eleven dimensional M theory in a continuous manner as the radius of $K3$ is taken to infinity. Although we have not yet worked out the details of heterotic matrix theory on higher dimensional tori, we think that it is clear that the infinite $K3$ limit will be one in which the χ degrees decouple from low energy dynamics.

The lesson we learn from this example is that *decompactification of space time dimensions leads to a reduction in degrees of freedom*. Indeed, this principle is clearly evident in the prescription for compactification of M theory on tori in terms of SYM theory. The more dimensions we compactify, the higher the dimension of the field theory we need to describe the compactification. There has been some discussion of whether this really corresponds to adding degrees of freedom since the requisite fields arise as limits of finite matrices. However there is a way of stating the principle which is independent of how one chooses to view these constructions. Consider, for example, a graviton state in M theory compactified on a circle. Choose a reference energy E and ask how the number of degrees of freedom with energy less than E which are necessary to describe this state, changes with the radius of compactification. As the radius is increased, the radius of the dual torus decreases. This decreases the number of states in the theory with energy less than E, precisely the opposite of what occurs when we increase the radius of compactification of a local field theory

4.1 Cosmological constant problem

It seems natural to speculate that this property, so counterintuitive from the point of view of local field theory, has something to do with the cosmological constant problem. In [13] one of the authors suggested that any theory which satisfied the 't Hooft-Susskind holographic principle would suffer a thinning out of degrees of freedom as the universe expanded, and that this would lead to an explanation of the cosmological constant problem. Although the speculations there did not quite hit the mark, the present ideas suggest a similar mechanism. Consider a hypothetical state of the matrix model corresponding to a universe with some number of Planck size dimensions and some other dimensions of a much larger size, R. Suppose also that SUSY is broken at scale B, much less than the (eleven dimensional) Planck scale. The degrees of freedom associated with the compactified dimensions all have energies much higher than the SUSY breaking scale. Their zero point fluctuations will lead to a finite, small (relative to the Planck mass) R independent, contribution to the total vacuum energy. As R increases, the number of degrees of freedom at scales less than or equal to B will decrease. Thus, we expect a corresponding decrease in the total vacuum energy. The total vacuum energy in the large R limit is thus bounded by a constant, and is dominated by the contribution of degrees of freedom associated with the small, compactified dimensions. Assuming only the minimal supersymmetric cancellation in the computation of the vacuum energy, we expect it to be of order $B^2 l_{11}$. This implies a vacuum energy density of order $B^2 l_{11}/R^3$, which is too small to be of observational interest for any plausible values of the parameters. If a calculation of this nature turns out to be correct, it would constitute a prediction that the cosmological constant is essentially zero in the matrix model.

It should not be necessary to emphasize how premature it is to indulge in speculations of this sort (but we couldn't resist the temptation). We do not understand supersymmetry breaking in the matrix model and we are even further from understanding its cosmology. Indeed, at the moment we do not even have a matrix model derivation of the fact[4] that parameters like the radius of compactification are dynamical variables. Perhaps the most important lacuna in our understanding is related to the nature of the large N limit. We know that many states of the system wander off to infinite energy as N is increased. Our discussion above was based on extrapolating results of the finite N models, without carefully verifying that the degrees of freedom involved survive the limit. Another disturbing thing about our discussion is the absence of a connection to Bekenstein's area law for the number of states. The Bekenstein law seems to be an integral part of the physical picture of the matrix model. Despite these obvious problems, we feel that it was worthwhile to present this preliminary discussion of the cosmological constant problem because it makes clear that the spacetime picture which will eventually emerge from the matrix model is certain to be very different from the one implicit in local field theory.

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